TECHNISCHE UNIVERSITÄT DRESDEN

Institute for Static und Dynamics of Structures

# Fuzzy Finite Element Method

# –

# FFEM

Bernd Möller

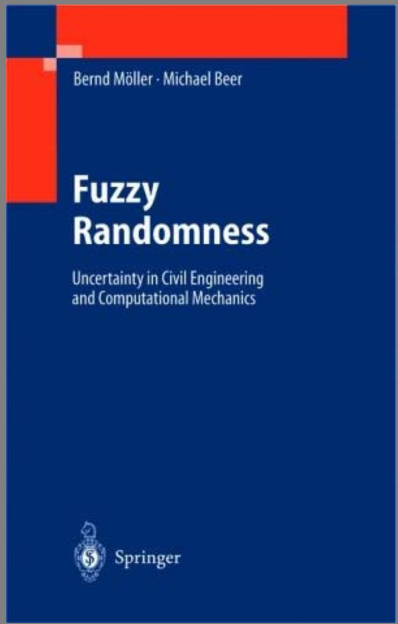
Recent research results about
non-classical methods in uncertainty modeling

**http://www.uncertainty-in-engineering.net**

and

**Fuzzy Randomness**

B. Möller, M. Beer,
Springer 2004

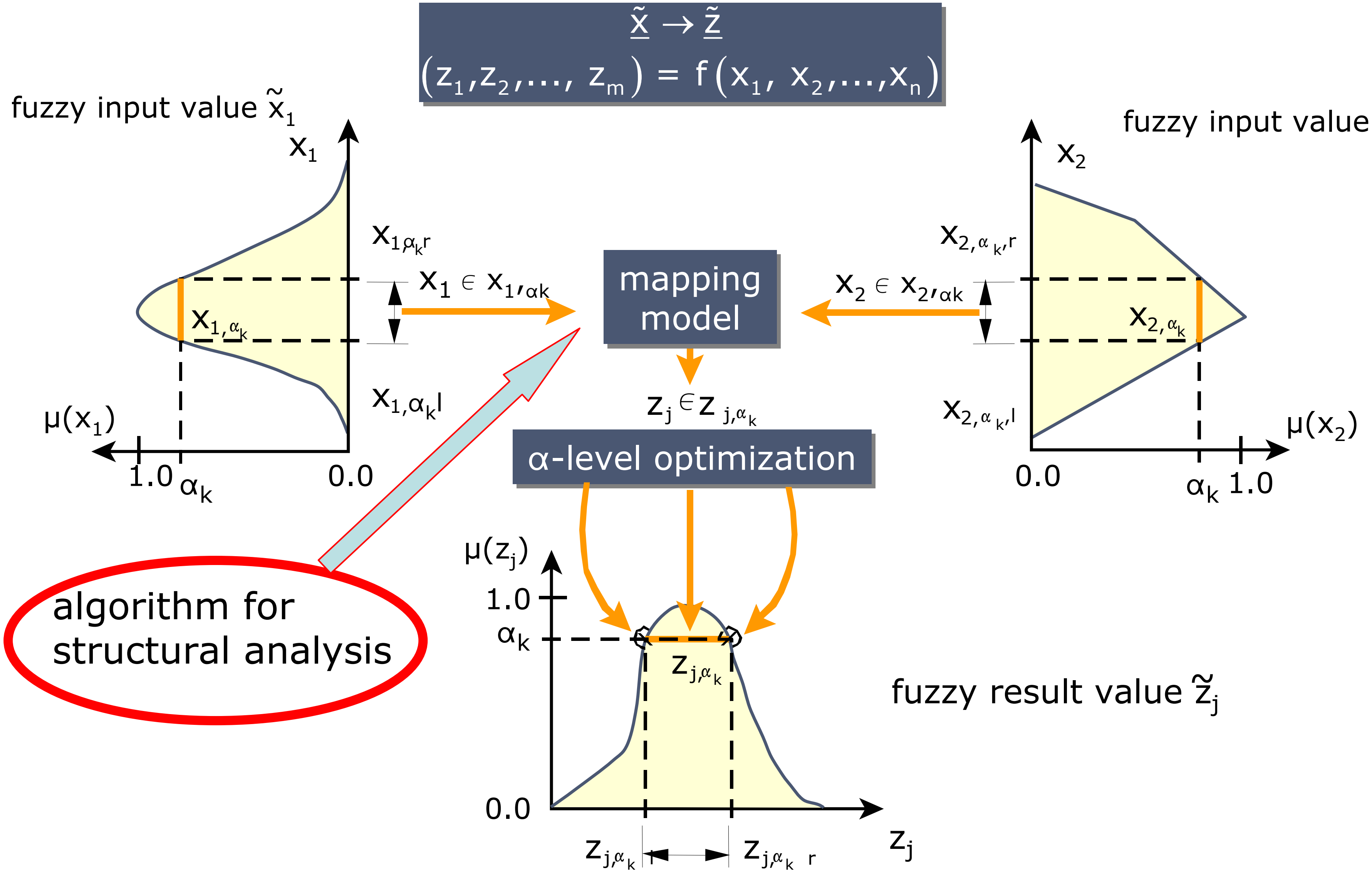


# Fuzzy Structural Analysis

**from fuzzy analysis to**
**fuzzy structural analysis**

# Fuzzy Structural Analysis





$$\tilde{x} \rightarrow \tilde{z}$$

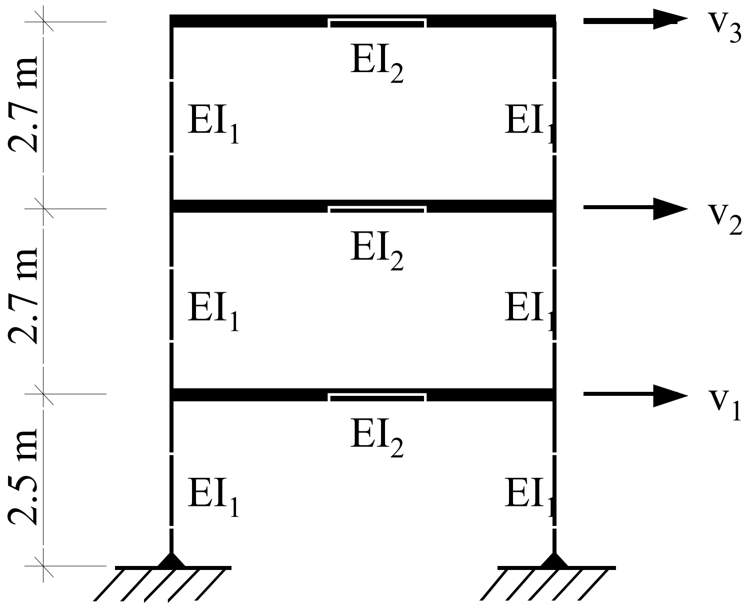
- Computing of fuzzy result values by means of a mapping model

$$z=(z_1;\ldots;z_j;\ldots z_m) = f(x_1;\ldots;x_i\,;\ldots;x_n)$$

- **f(x) represents the mapping model M = structural analysis**
- Parameter of the model M might also be uncertain
- The mapping model becomes uncertain

$$\tilde{f}= M(\tilde{m}_1;\ldots;\tilde{m}_r\,;\ldots;\tilde{m}_p)$$

## Vibration analysis of a multistory frame




$EI_2 \rightarrow \infty; \quad EA \rightarrow \infty$ (bending and extensional stiffnesses)

**fuzzy differential equation system**

$$\frac{d\tilde{\underline{x}}}{dt} = \tilde{\underline{A}} \cdot \tilde{\underline{x}}\,; \quad \tilde{\underline{x}} = \begin{bmatrix} \tilde{\underline{v}} \\ \dot{\tilde{\underline{v}}} \end{bmatrix}$$

Cauchy problem with fuzzy initial conditions and fuzzy coefficients

- masses are concentrated in the horizontal bars
- prescribed initial value for the velocity of the upper horizontal bar

## deterministic parameters

$$EI_1 = 4 \cdot 10^3 \text{ kNm}^2$$

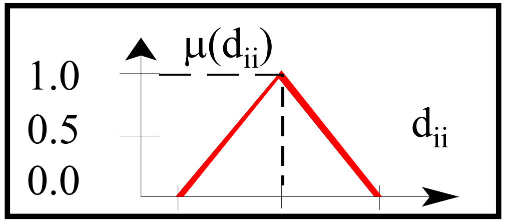
## fuzzy damping parameters

$$\tilde{d}_{11} = \langle 0.045; 0.05; 0.060 \rangle \text{ kNsm}^{-1}$$

$$\tilde{d}_{22} = \langle 0.040; 0.05; 0.055 \rangle \text{ kNsm}^{-1}$$

$$\tilde{d}_{33} = \langle 0.045; 0.05; 0.060 \rangle \text{ kNsm}^{-1}$$

$$\underline{\tilde{D}} = \begin{bmatrix} \tilde{d}_{11} & 0 & 0 \\ 0 & \tilde{d}_{22} & 0 \\ 0 & 0 & \tilde{d}_{33} \end{bmatrix}$$




## fuzzy initial conditions

$$\underline{v}_0 = \{ 0; 0; 0 \}; \; \underline{\tilde{\dot{v}}}_0 = \{ 0; 0; \tilde{\dot{v}}_{30} \}$$

$$\tilde{\dot{v}}_{30} = \langle 0.9; 1.0; 1.2 \rangle \text{ ms}^{-1}$$

## mapping operator

$$\underline{z} = f(\underline{x}) = \underline{v}(\underline{D}; \dot{v}_{30}; t) = \begin{bmatrix} v_1(\underline{D}; \dot{v}_{30}; t) \\ v_2(\underline{D}; \dot{v}_{30}; t) \\ v_3(\underline{D}; \dot{v}_{30}; t) \end{bmatrix}$$

# Fuzzy Structural Analysis – Dynamic (1)



$$(\tilde{d}_{11};\tilde{d}_{22};\tilde{d}_{33};\dot{\tilde{v}}_0(3);t) \longrightarrow (\tilde{v}(1);\tilde{v}(2);\tilde{v}(3);\dot{\tilde{v}}(1);\dot{\tilde{v}}(2);\dot{\tilde{v}}(3))$$

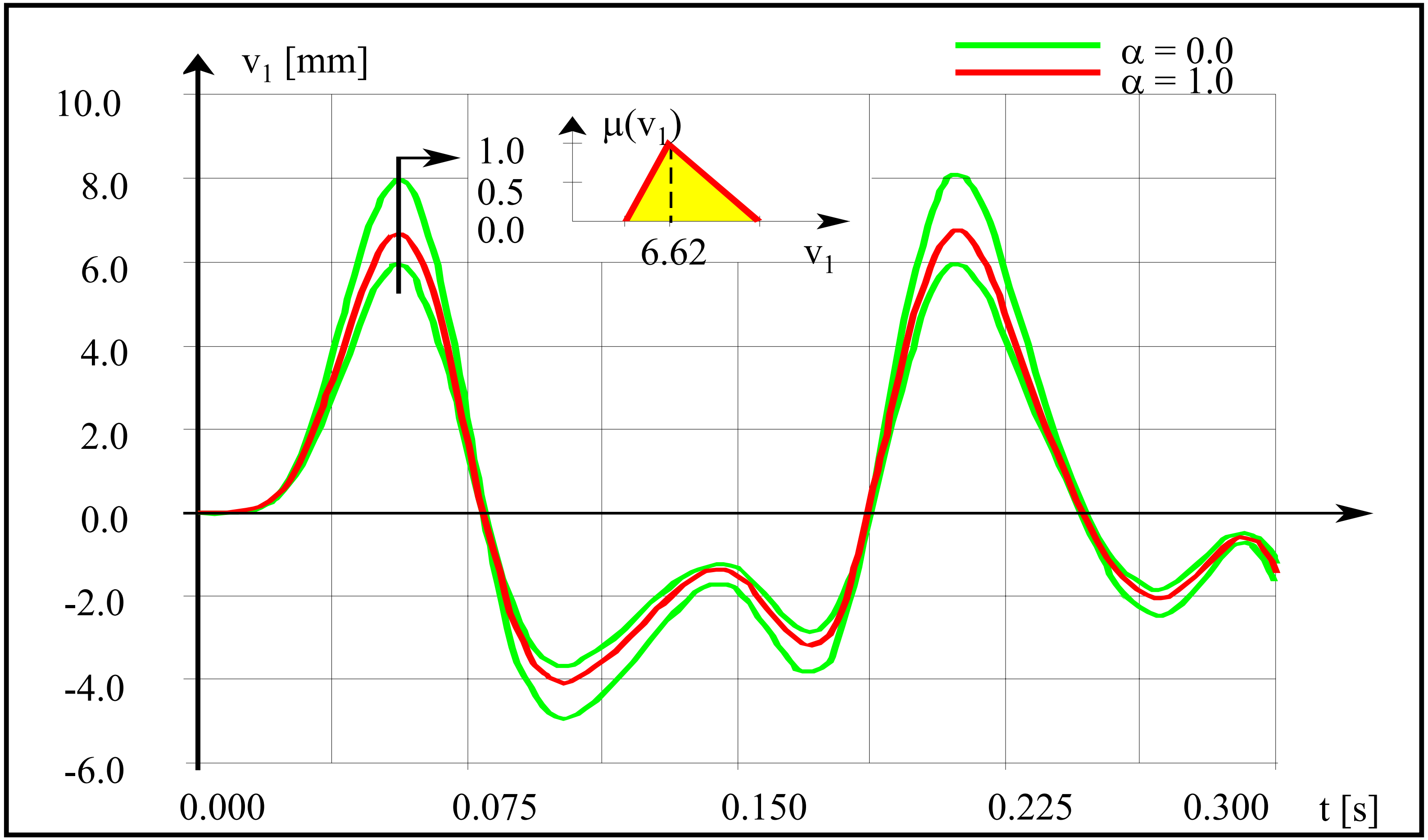
four-dimensional
fuzzy input space,
also depending from
the crisp parameter t

six-dimensional
fuzzy result space

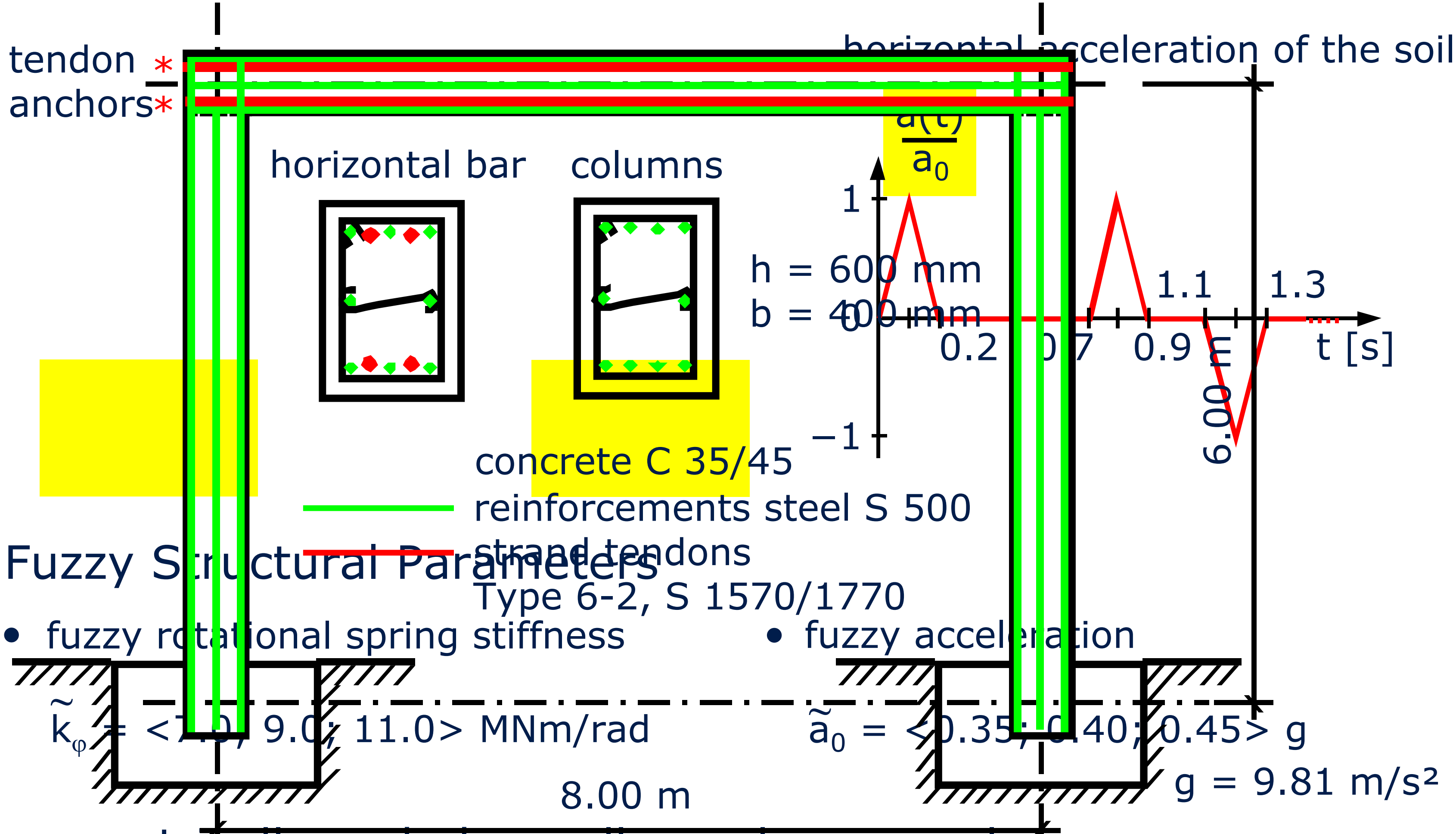
mapping is not biunique

no monotonicity

## fuzzy displacement-time dependence of the lowest story



## Prestressed Reinforced Concrete Frame (prefabricated segments)




## Fuzzy Structural Parameters

- fuzzy rotational spring stiffness

  $\tilde{k}_{\varphi} = <7.0;\ 9.0;\ 11.0>$ MNm/rad

- fuzzy acceleration

  $\tilde{a}_0 = <0.35;\ 0.40;\ 0.45>$ g

  g = 9.81 m/s²

## geometrically and physically nonlinear analysis

# Fuzzy Structural Analysis – Dynamic (2)



System modification (during montage) and loading process

1. Simultaneous prestressing of all tendons in the horizontal bar without the effects of deadload

1. Application of the deadload and hinged connection of the columns and the horizontal bar

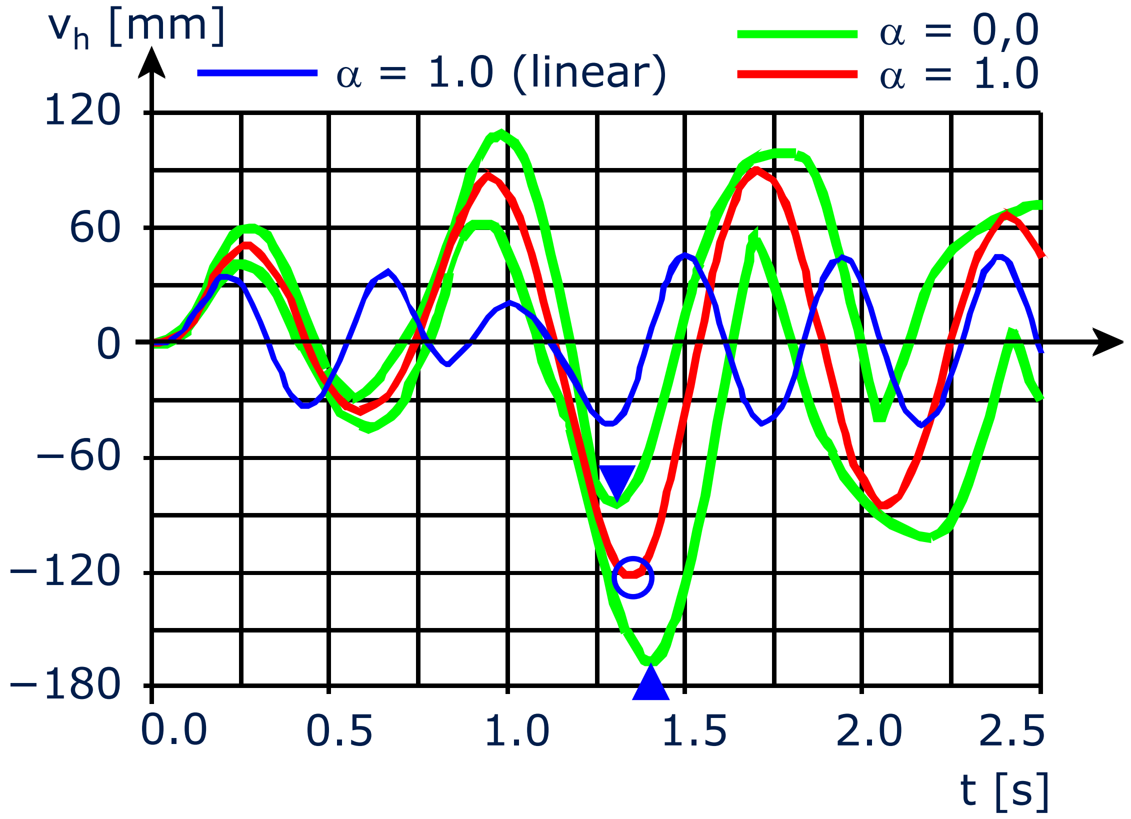
3. Transformation of the hinged joints at the corners into rigid connections

4. Application of additional translation mass at the frame corner

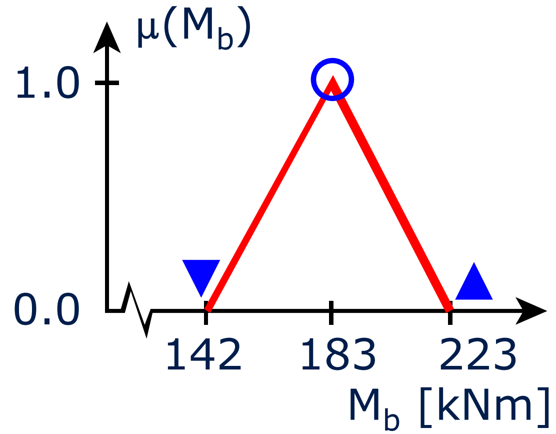
5. Introduction of dynamic loading due to the horizontal acceleration

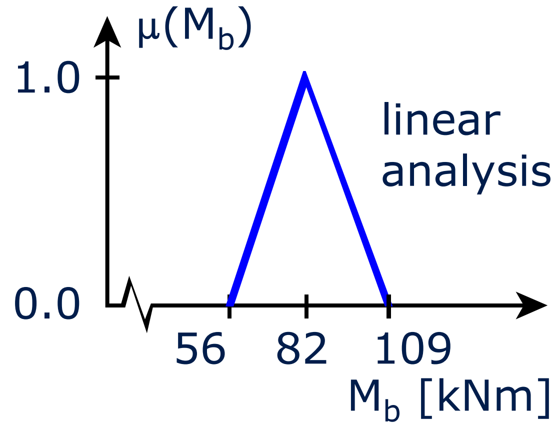
## Fuzzy Structural Responses (nonlinear analysis)

- horizontal displacement $\tilde{v}_h(t)$ (left-hand frame corner)




- maximum end-fixing moment $\tilde{M}_b$ (right-hand column base)



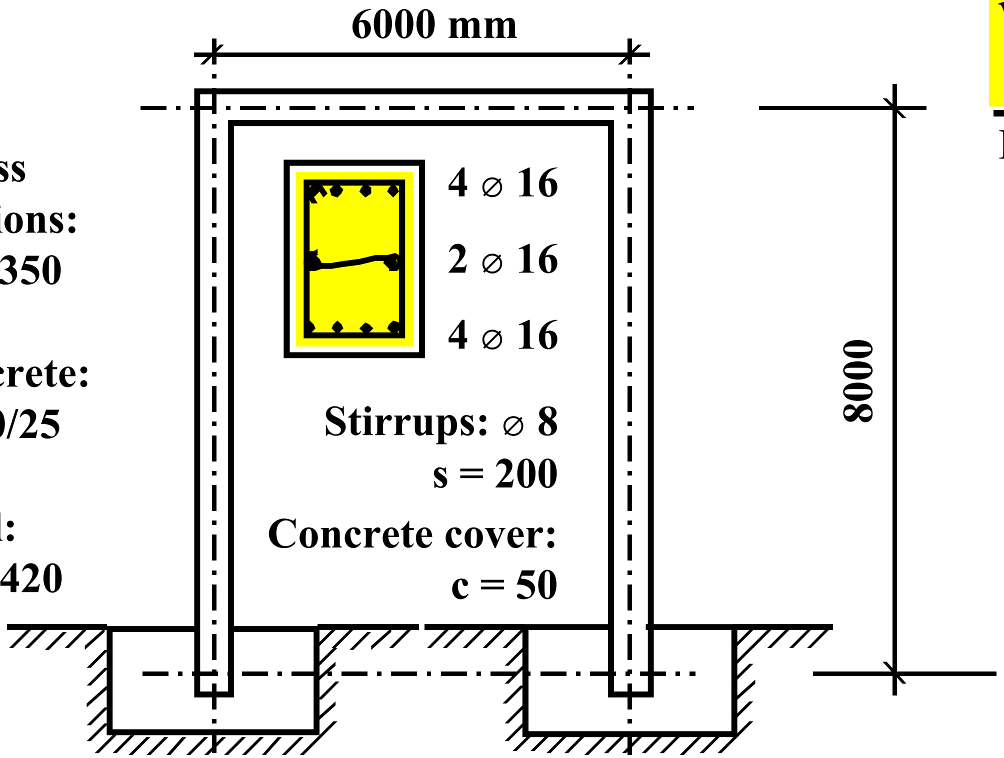
# Reinforced-Concrete Frame - Static

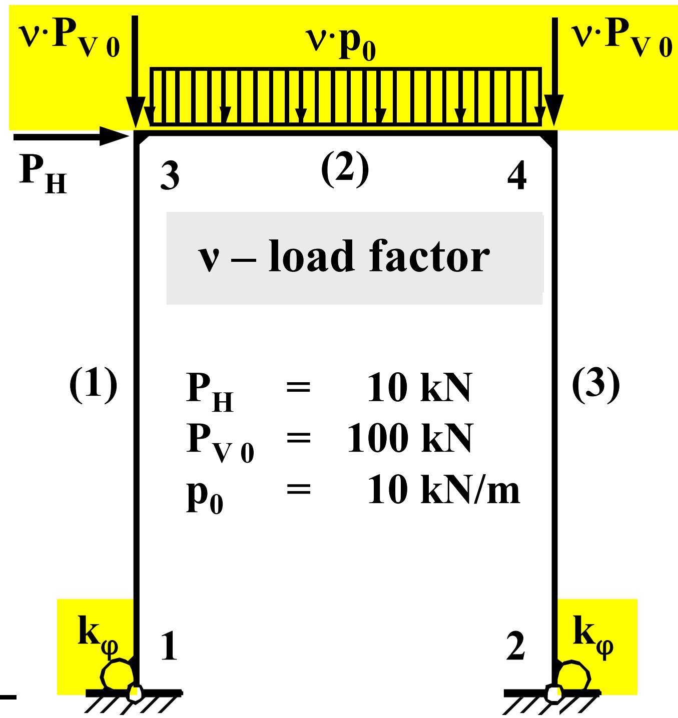


**Cross sections:**
**500/350**

**concrete:**
**C 20/25**

**steel:**
**BSt 420**



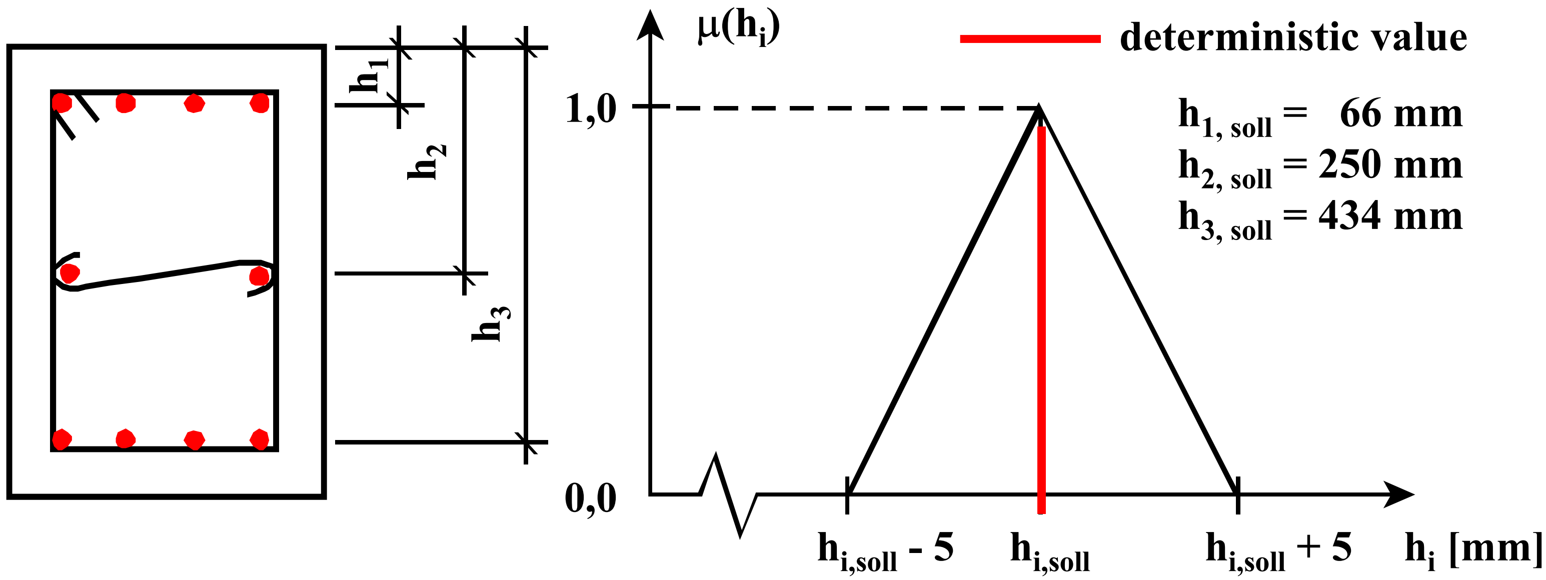

- **Loading :**
  1) **dead load**
  2) **horizontal load $P_H$**
  3) **vertical load $\nu \cdot P_{V\,0}$ and $\nu \cdot p_0$** (increasing of ν until system failure)

geometrically and physically nonlinear analysis

## Fuzzy arrangement of the reinforcement steel

- Fuzzy distances $\tilde{h}_i$ at each end of the bars and in the middle of horizontal bar:

**21 fuzzy triangle numbers**



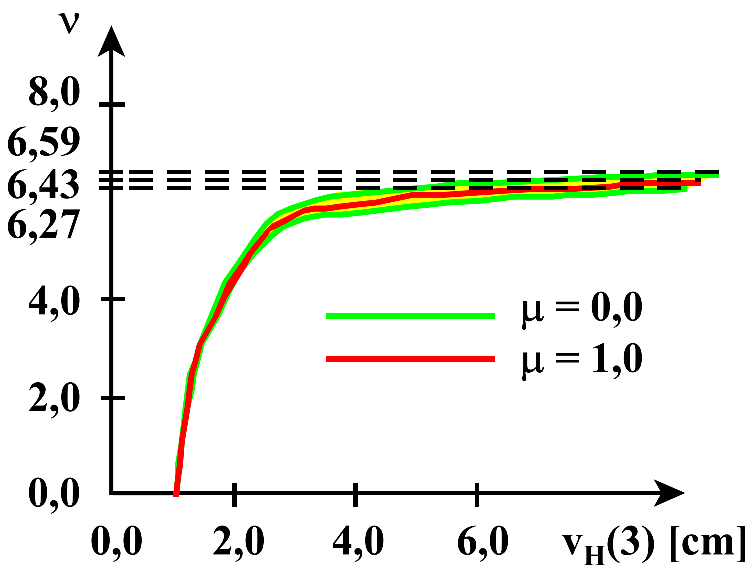

- deterministic stiffness of the rotational spring $k_\varphi$ = 5 MNm/rad
- loading up to global system failure

# Reinforced-Concrete Frame - Analysis (1)

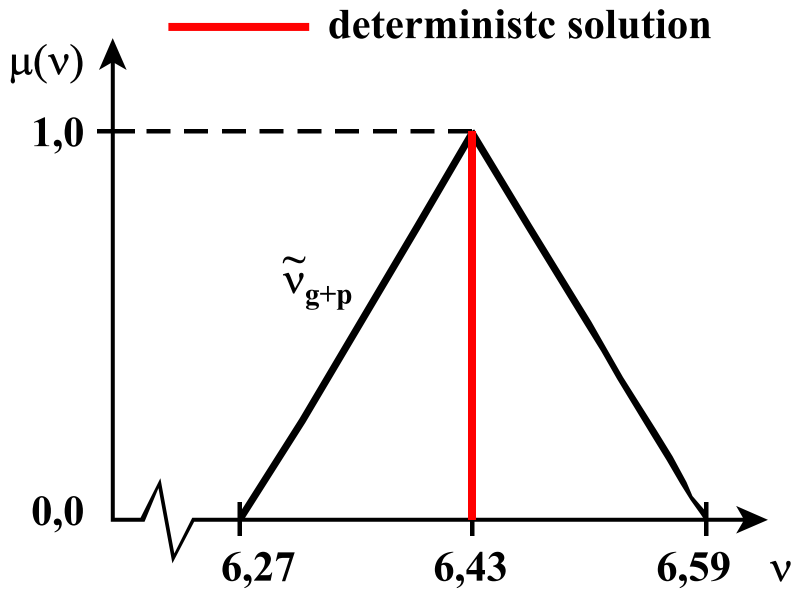


## Fuzzy results

- **Fuzzy load-displacement dependency (left corner, horizontal)**




- **Fuzzy failure load: load factor**



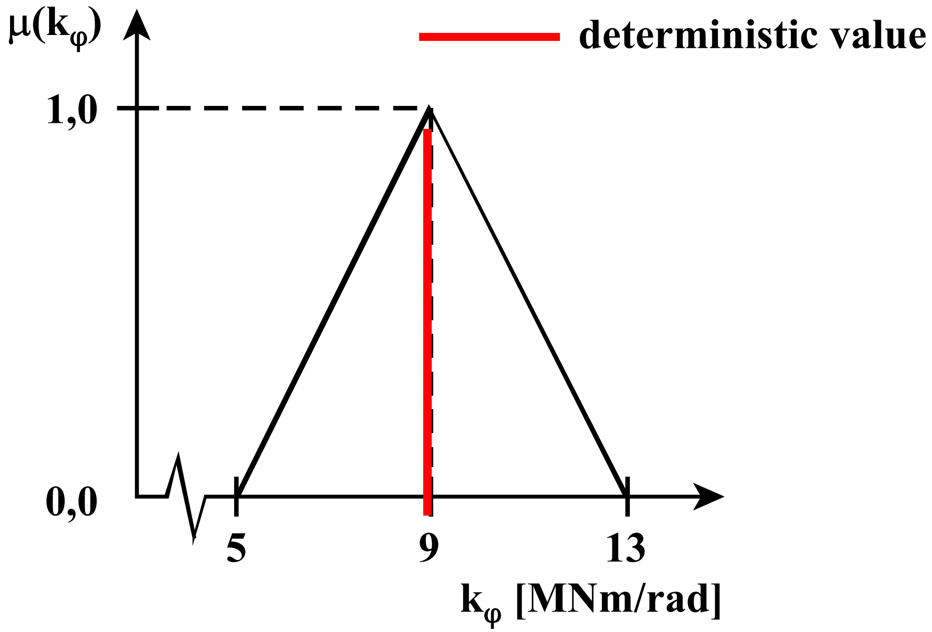
## Fuzzy input data:

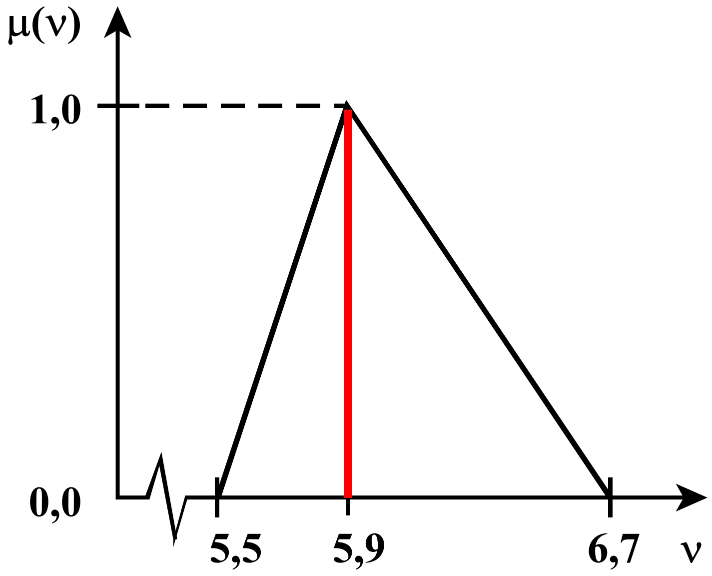
- **Fuzzy rotational spring stiffness**

  $\tilde{k}_{\varphi} = < 5;\ 9;\ 13 >$ **MNm/rad**

- **Fuzzy load factor (maximal)**

  $\tilde{\nu} = < 5{,}5;\ 5{,}9;\ 6{,}7 >$



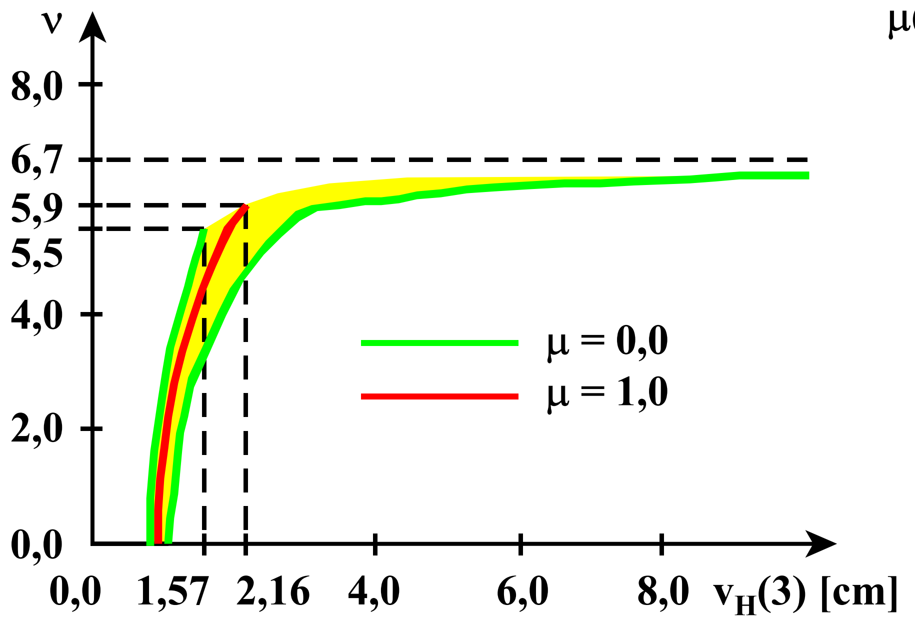

- **Reinforcement arrangement is deterministically !!**

# Reinforced-Concrete Frame - Analysis (2)

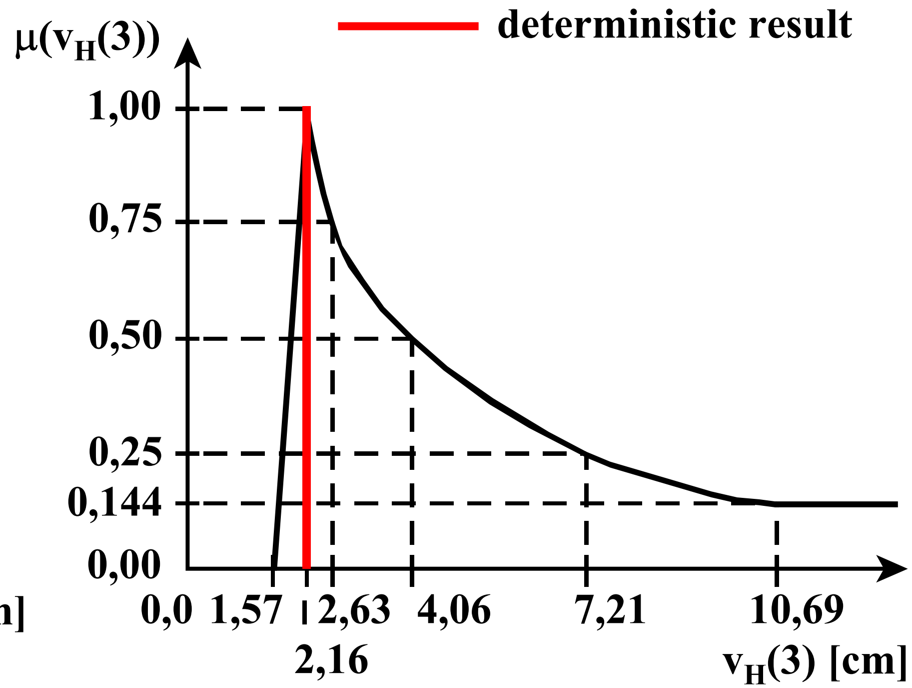


## Fuzzy results

- **Fuzzy load-displacement dependency (left corner, horizontal)**




- **Fuzzy displacement (failure state)**




investigation of six α-level

# Fuzzy Finite Element Method
# –
# FFEM

**from fuzzy analysis to**
**fuzzy finite element analysis**

**Assumption:**

- structures in $\mathbb{R}^1$ and $\mathbb{R}^2$
- position vector $\underline{\theta} = \{\theta_1, \theta_2, \tilde{\theta}_3\}$

**1 Displacement field $\underline{\tilde{v}}(\underline{\theta})$ containing fuzziness is chosen**

$$\underline{\tilde{v}}(\underline{\theta}) = \underline{N}(\underline{\theta}) \bullet \underline{\tilde{v}}(e); \quad \underline{\theta} = \{\theta_1, \theta_2\}$$

**2. Linear relationship between generalized strains and displacements**

$$\underline{\tilde{\varepsilon}}(\underline{\theta}) = \underline{H}(\underline{\theta}) \bullet \underline{\tilde{v}}(e)$$

**3. Linear material law**

$$\underline{\tilde{\sigma}}(\underline{\theta}) = \underline{\tilde{E}}(\underline{\theta}) \bullet \underline{\tilde{\varepsilon}}(e) = \underline{\tilde{E}}(\underline{\theta}) \bullet \underline{H}(\underline{\theta}) \bullet \underline{\tilde{v}}(e)$$

### 4. Virtuel displacements, virtual strains

$$\delta\underline{\tilde{v}}(\underline{\theta}) = \underline{N}(\underline{\theta})\bullet\delta\underline{\tilde{v}}(e)$$

$$\delta\underline{\tilde{\varepsilon}}(\underline{\theta}) = \underline{H}(\underline{\theta})\bullet\delta\underline{\tilde{v}}(e)$$

### 5. Virtuel internal fuzzy work

$$\delta\tilde{A}_i = \int_{\tilde{V}} \delta\underline{\tilde{\varepsilon}}^T(\underline{\theta})\bullet\underline{\tilde{\sigma}}(\underline{\theta})\, d\tilde{V}$$

$$\delta\tilde{A}_i = \delta\underline{\tilde{v}}^T(e)\bullet\int_{\tilde{V}} \underline{H}^T(\underline{\theta})\bullet\underline{\tilde{E}}(\underline{\theta})\bullet\underline{H}(\underline{\theta})\, d\tilde{V}\bullet\underline{\tilde{v}}(e)$$

Fuzzy element stiffness matrix $\underline{\tilde{K}}(e)$

## 6. virtuel external fuzzy work

$$\delta\tilde{A}_a = \delta\underline{\tilde{v}}^T(e) \bullet \underline{\tilde{F}}(e, t)$$

virtuel work of nodal forces

$$+ \int_{\tilde{V}} \delta\underline{\tilde{v}}^T(\underline{\theta}) \bullet \underline{\tilde{p}}_M(\underline{\theta})\, d\tilde{V}$$

virtuel work of mass forces

$$+ \int_{O} \delta\underline{\tilde{v}}^T(\underline{\theta}) \bullet \underline{\overset{+}{\tilde{p}}}(\underline{\theta}, t)\, dA_O$$

virtuel work of time-dependent surface forces

$$+ \int_{\tilde{V}} \delta\underline{\tilde{v}}^T(\underline{\theta}) \bullet \tilde{\rho}(\underline{\theta}) \bullet \underline{\ddot{\tilde{v}}}(\underline{\theta}, t)\, d\tilde{V}$$

virtuel work of inertial forces

$$+ \int_{\tilde{V}} \delta\underline{\tilde{v}}^T(\underline{\theta}) \bullet \underline{\tilde{d}}(\underline{\theta}) \bullet \underline{\dot{\tilde{v}}}(\underline{\theta}, t)\, d\tilde{V}$$

virtuel work of damping forces

**Fuzzy differential equation of second order**

$$\underline{\tilde{M}} \bullet \underline{\tilde{\ddot{v}}} + \underline{\tilde{D}} \bullet \underline{\tilde{\dot{v}}} + \underline{\tilde{K}} \bullet \underline{\tilde{v}} = \underline{\tilde{F}}$$

**in the static case:**

$$\underline{\tilde{K}} \bullet \underline{\tilde{v}} = \underline{\tilde{F}}$$

# Solution technique for time-independent problems



given: $\underline{\tilde{P}}_j\left(\underline{\theta}, \underline{\tilde{s}}_j\right)$  $\mathbf{j = 1, ..., n_{FF}}$  **fuzzy – functions**

$\underline{\tilde{s}}_j = \left\{\tilde{\mathbf{s}}_{j,1}, \tilde{\mathbf{s}}_{j,2}, ..., \tilde{\mathbf{s}}_{j,r}\right\}$  **fuzzy bunch parameter vectors with $\tilde{s}_{j,r}$ fuzzy bunch parameters**

- $\underline{\tilde{v}} = \underline{K}^{-1}\left(..., \tilde{s}_{j,r}, ...\right) \bullet \underline{F}\left(..., \tilde{s}_{j,r}, ...\right)$
- all fuzzy bunch parameters possess the same of $\alpha$-levels
- each trajectory describes one displacement field $\underline{v}(\underline{\theta}) = f\left(..., \tilde{s}_{j,r}, ...\right)$
- computing the membership functions of the fuzzy result values with $\alpha$-level optimization

# Bunch Parameter Representation of Fuzzy Functions



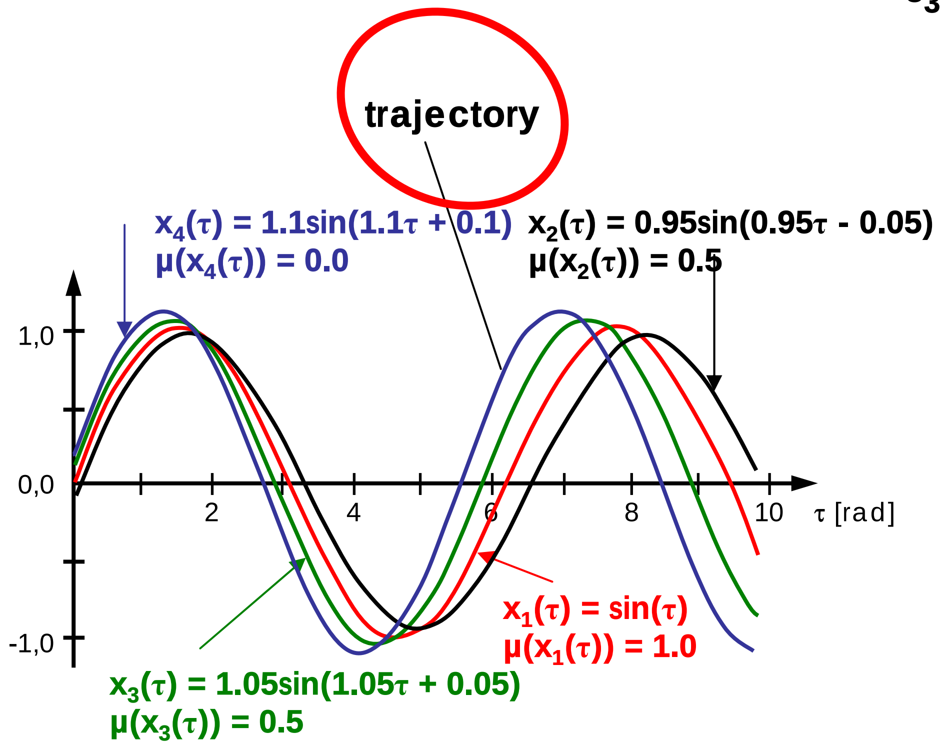
$\tilde{\underline{x}}(\underline{t}) = x(\tilde{\underline{s}}, \underline{t})$ (as recapitulation)

$\tilde{\underline{x}}(\tau) = x(\tilde{\underline{s}}, \tau) = \tilde{s}_1 \cdot \sin(\tilde{s}_2 \cdot \tau + \tilde{s}_3)$ with $\tilde{\underline{s}} = (\tilde{s}_1, \tilde{s}_2, \tilde{s}_3)$

$\tilde{s}_1 = < 0.9, 1.0, 1.1 >$

$\tilde{s}_2 = < 0.9, 1.0, 1.1 >$

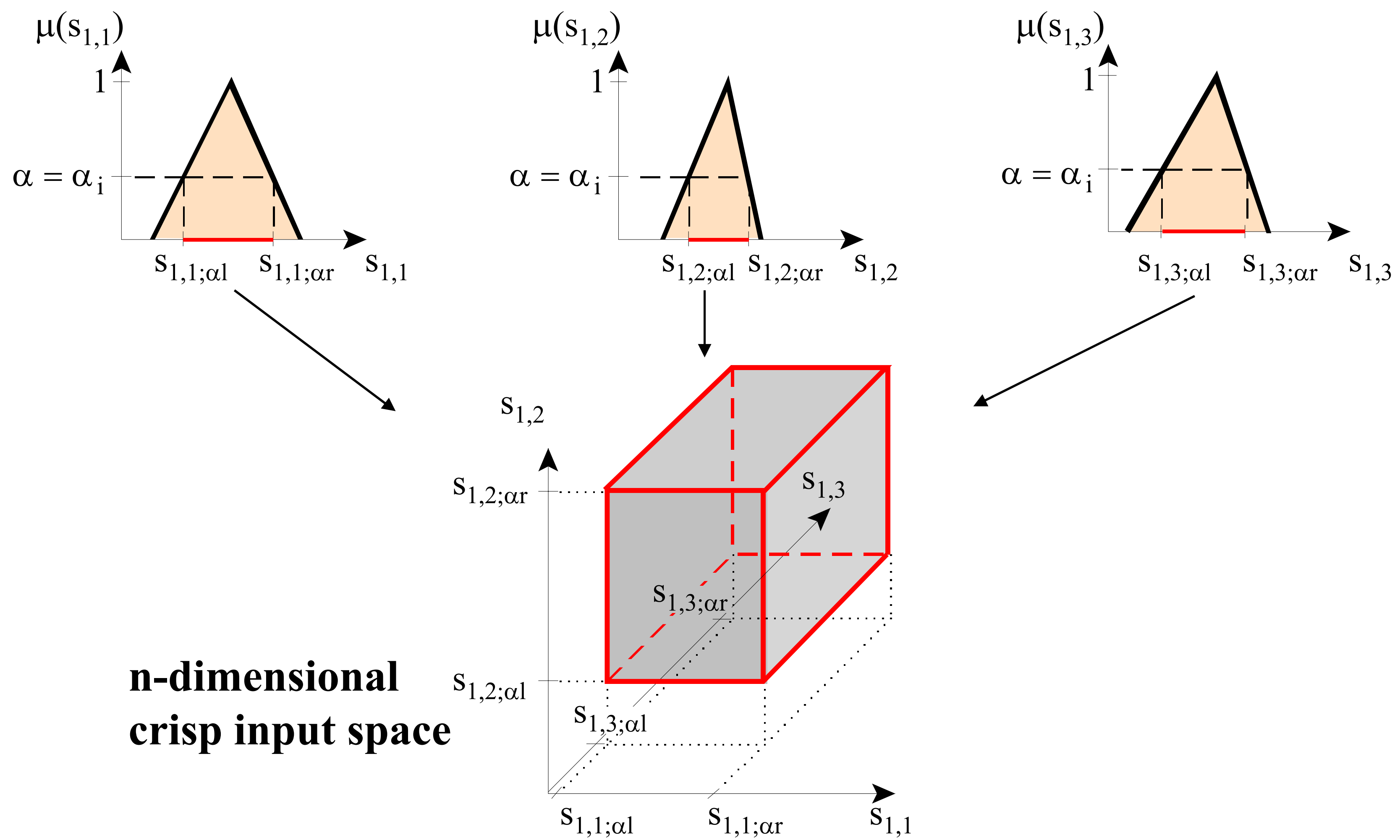
$\tilde{s}_3 = < -0.1, 0.0, 0.1 >$



# Solution technique for time-independent problems



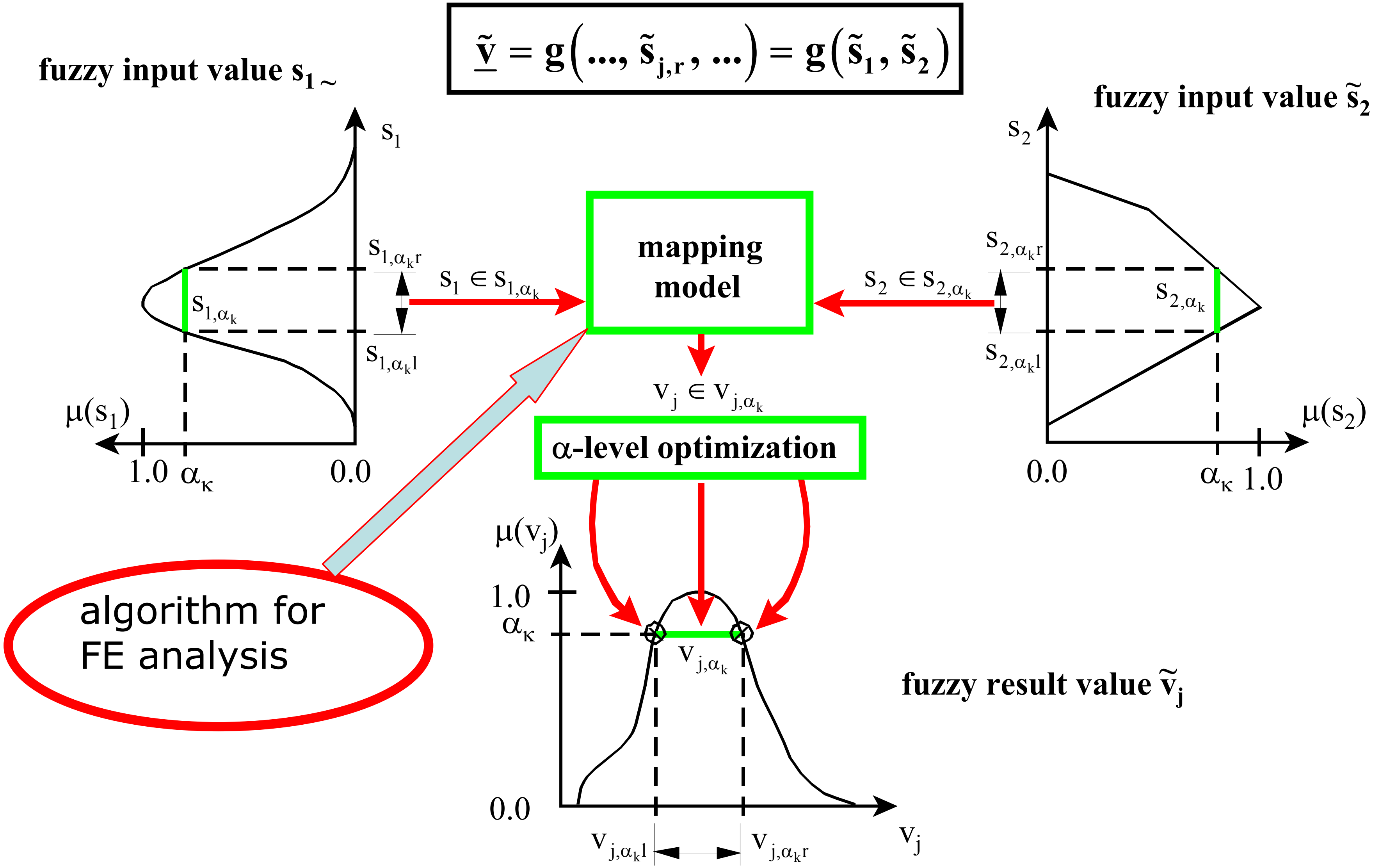
**Generation of uncertain input spaces (bunch parameter $\tilde{s}_{1,1}$, $\tilde{s}_{1,2}$, $\tilde{s}_{1,3}$)**




**n-dimensional crisp input space**

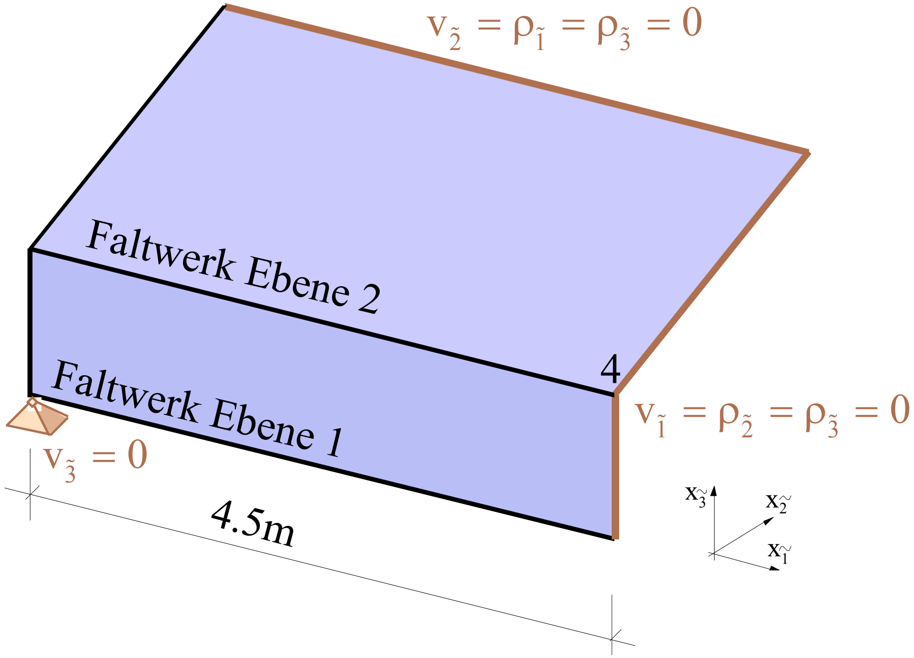
# Solution technique for time-independent problems

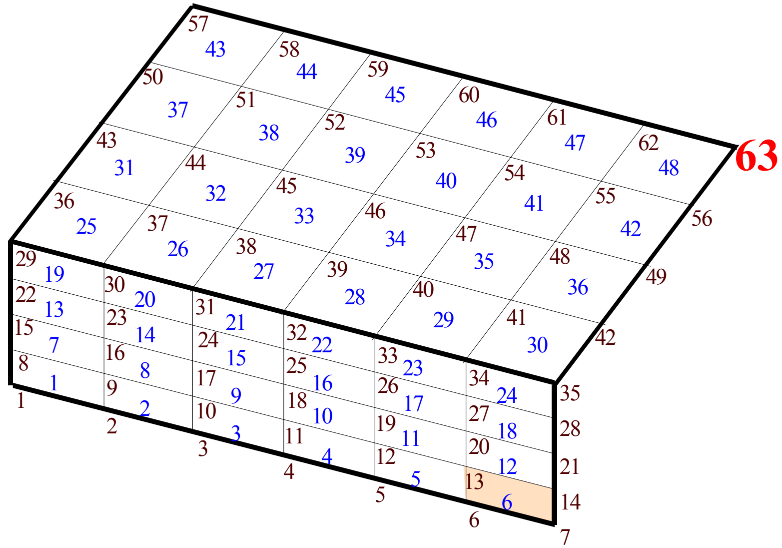




# Example: geometry and finite element model



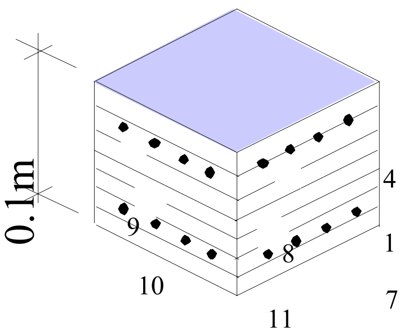


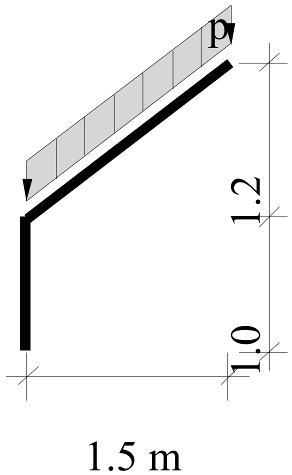




**concrete** C 20/25:
layer 1-7

reinforcement S 500:

| | |
|---|---|
| layer 8 | 7.85 cm²/m |
| layer 9 | 2.52 cm²/m |
| layer 10 | 2.52 cm²/m |
| layer 11 | 7.85 cm²/m |



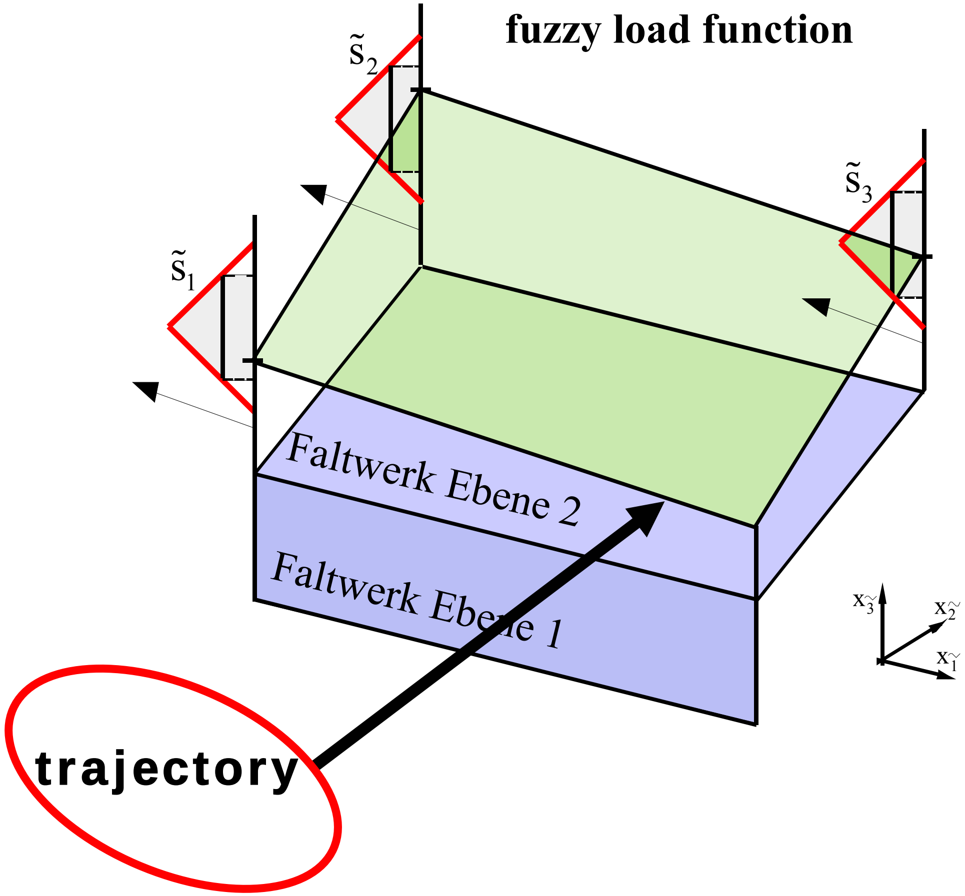




**reinforced concrete structure**

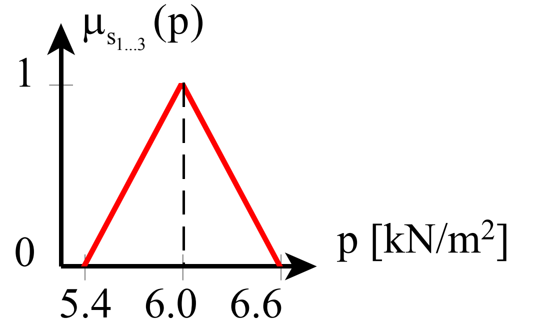
# FFEM-analysis (1)

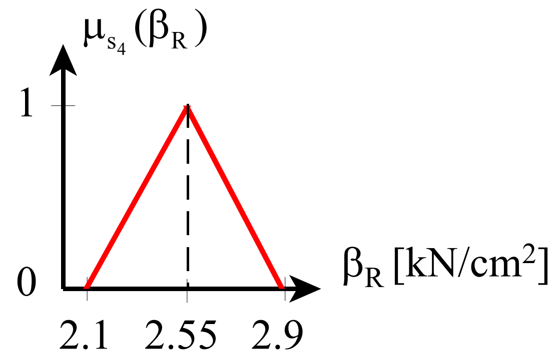


## fuzzy input values




**bunch parameter**



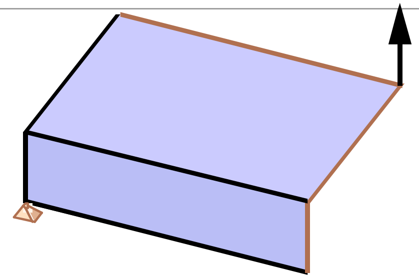



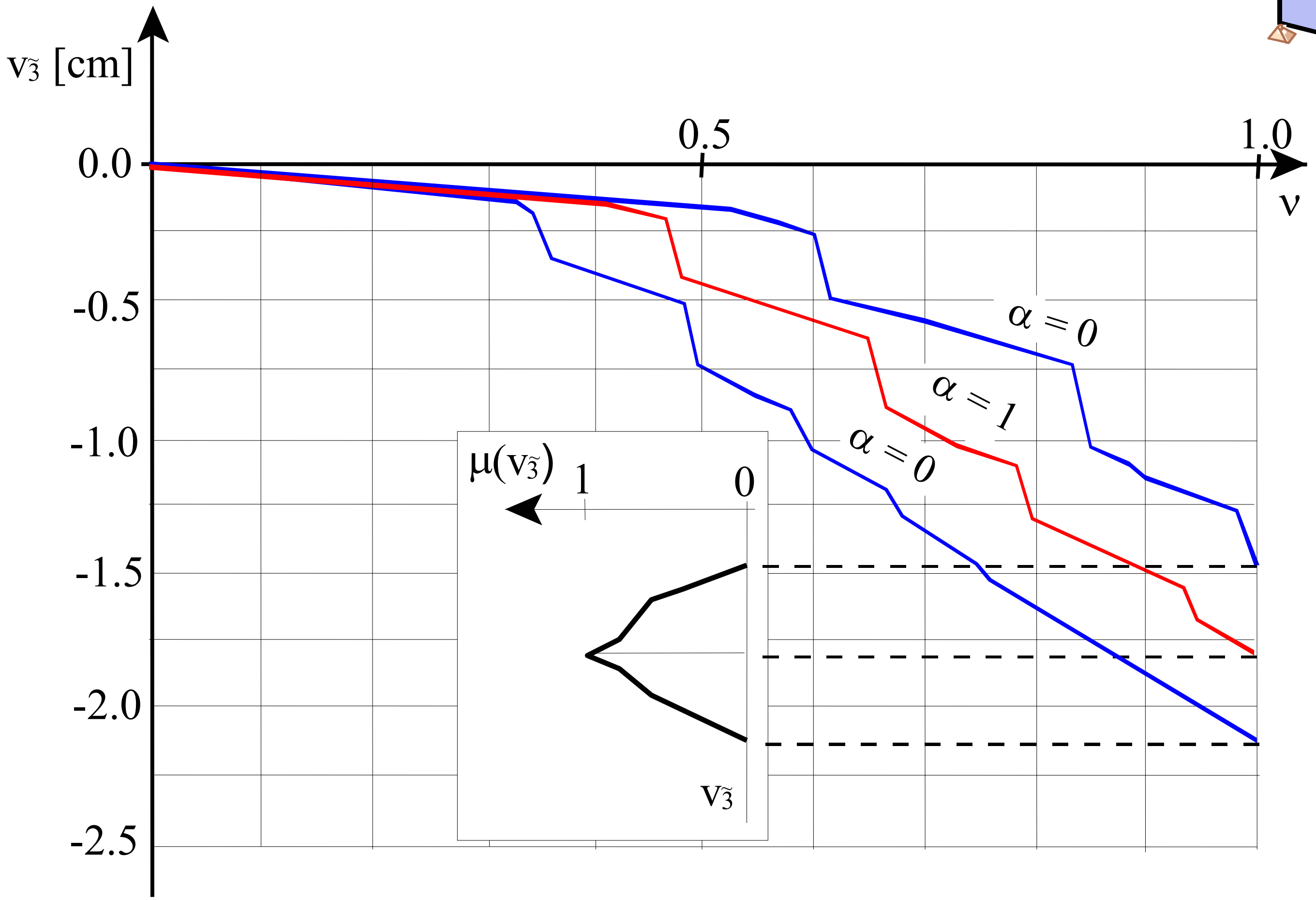

concrete compressive strength
= stationary fuzzy field

# FFEM-analysis (1)



**Fuzzy load-displacement dependency for $v_3$ (node 63)**

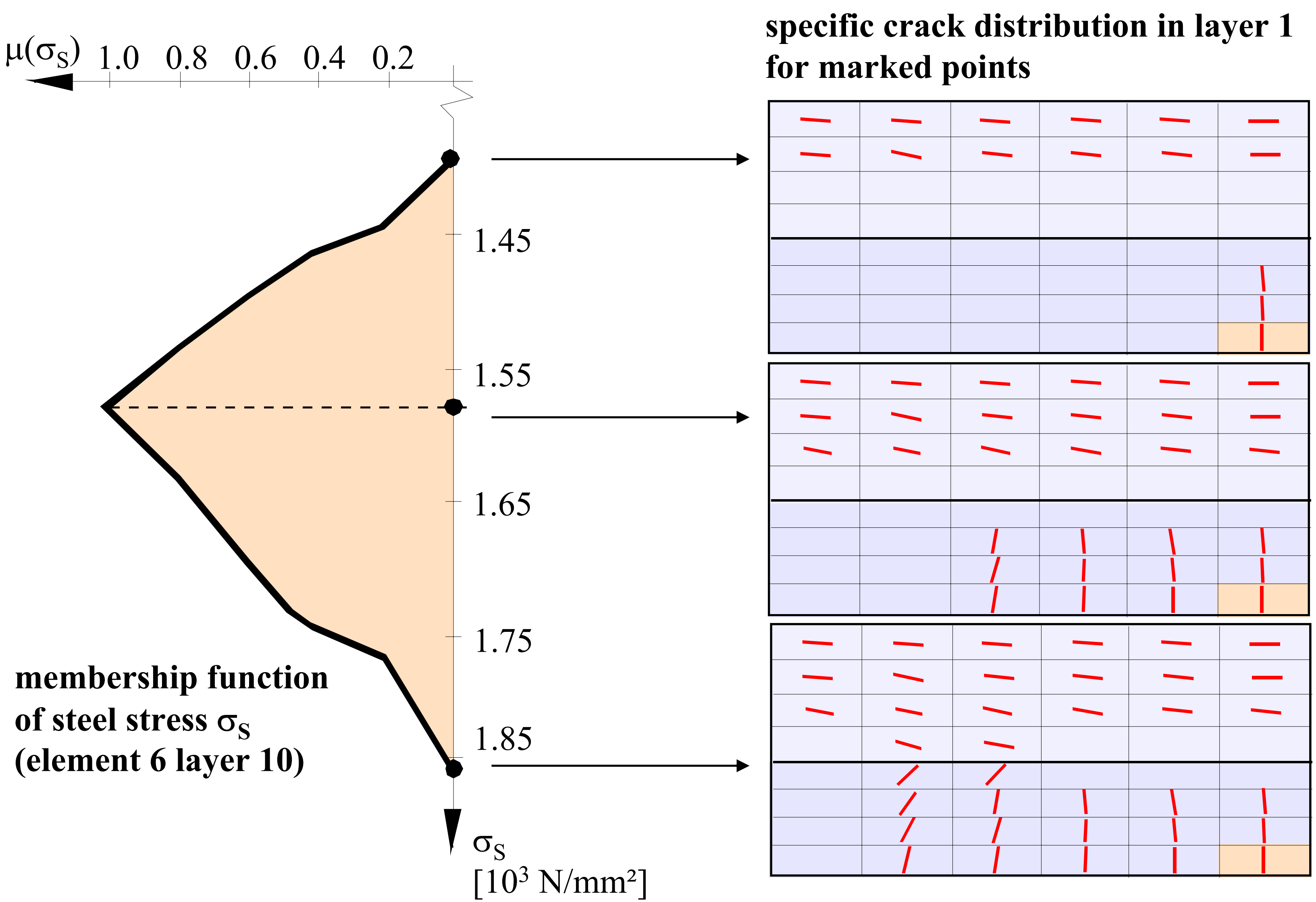





ν ... load factor

# FFEM-analysis (1)



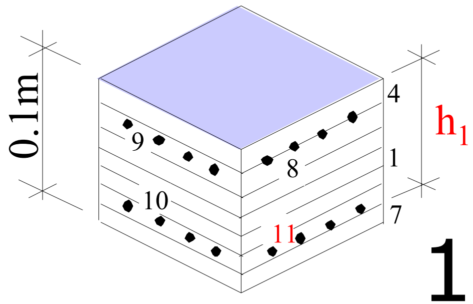



**membership function of steel stress $\sigma_S$ (element 6 layer 10)**

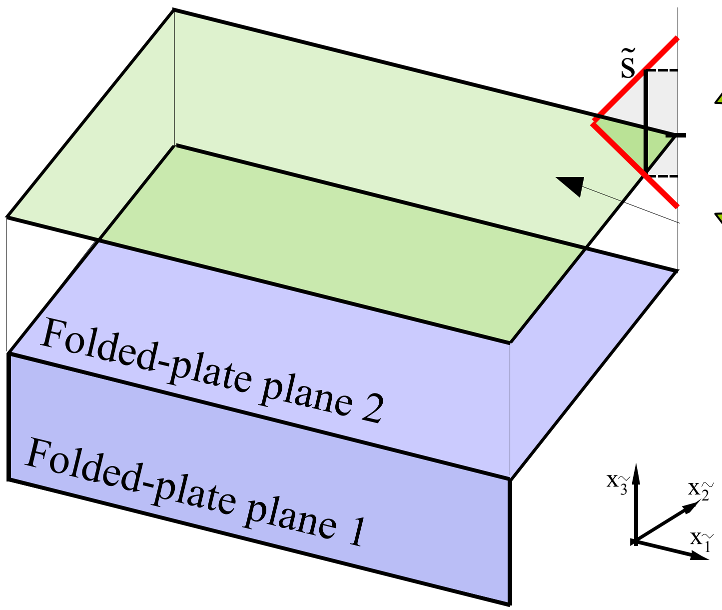
# FFEM-analysis (2)



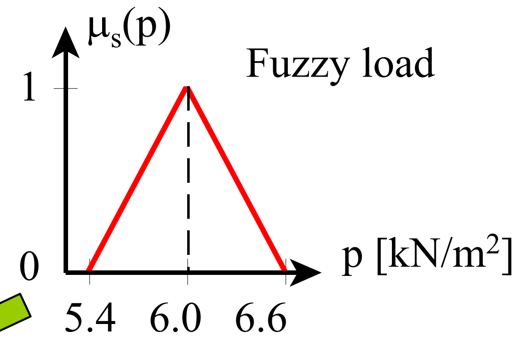
**fuzzy input values**



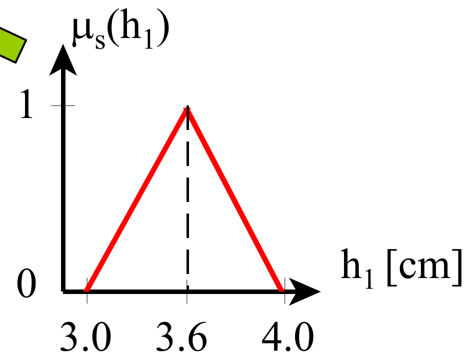

1 **fuzzy function with one bunch parameter**




**fuzzy bunch parameeter**




2 **fuzzy arrangement of reinforcement (only plane 2)**

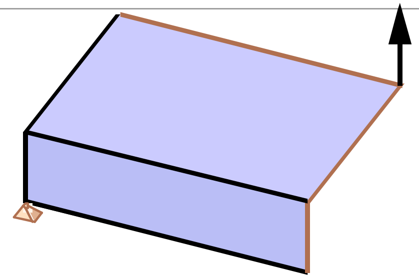


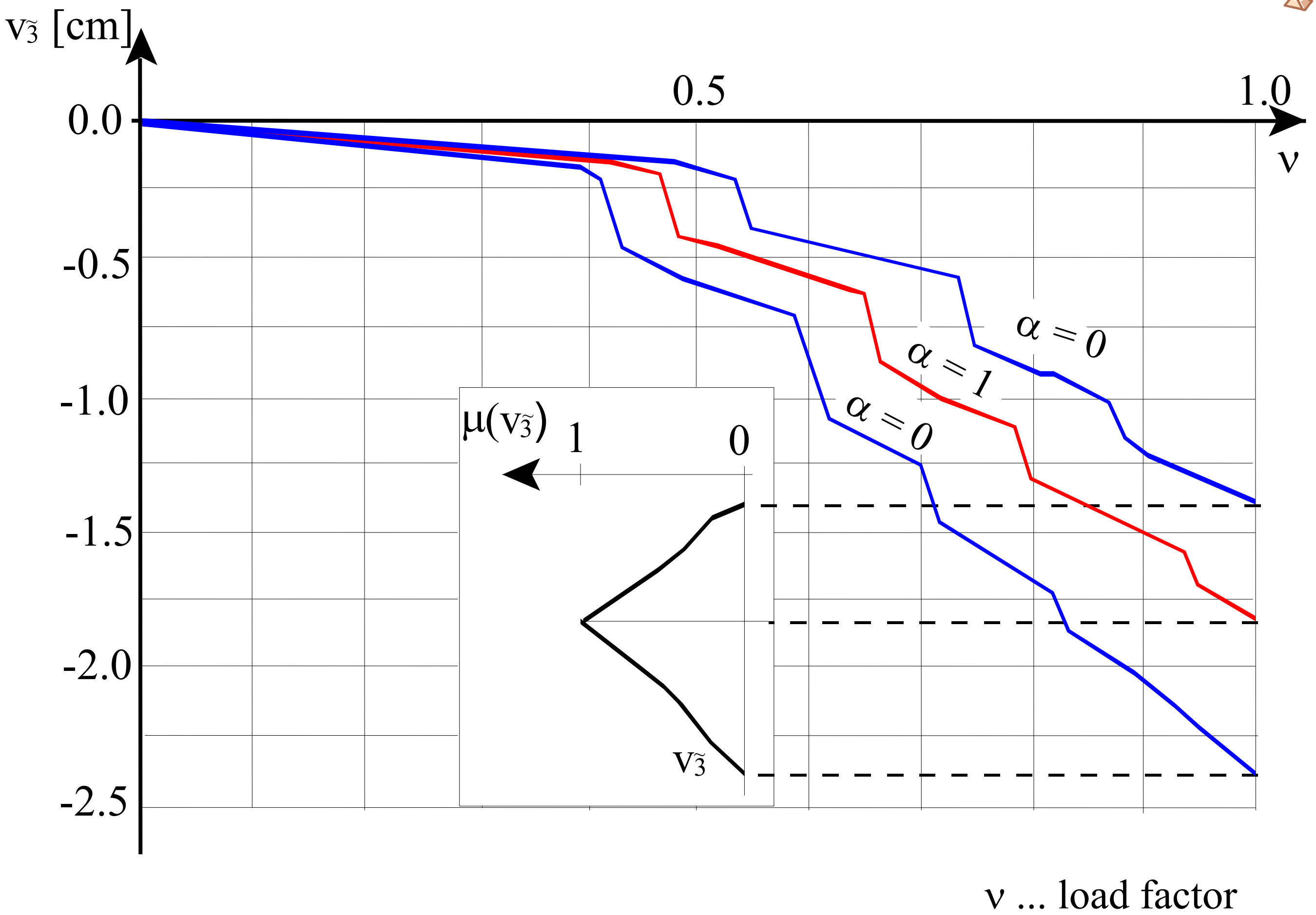

both: stationary fuzzy fields

# FFEM-analysis (2)



**Fuzzy load-displacement dependency for $v_3$ (node 63)**





Thank you !